Bell-Tower Dynamic Motion: Latest Research

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Acknowledgements:

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Introduction

1. What is the problem being considered?
2. Why are we studying this problem?
3. Nature of the problem
4. Theory
5. Static and dynamic testing at Wingrave church
6. Comparisons between theory and experiment
7. Using a bell to determine tower structural parameters
8. Conclusions

My challenge is to answer the questions the above and to explain the issues to a non-expert audience in one hour (in an entertaining way) without mathematics…..

…this is of course impossible, but I can try!

Maths is kept to a minimum, and used only to illustrate where/why/how things happen.

1/8. What is the problem being considered?

- When ringing a bell (or bells) in a bell tower, we want to be able to predict the amplitude of the tower response.

2/8. Why are we studying this problem?

- The motivation behind the work relates to bell handling.
- If a bell tower motion can be predicted, then the bell weights, orientations, and/or the heights of the bells can be optimised to minimise tower motion.
- There is a bell frame height movement criteria (attributed to Lewis) of 1/16th of an inch (0.062”, or 1.6×10^{-3}m, or 1.6mm).

3/8. Nature of the Problem

i. We need to predict the horizontal time-varying forces created by a bell when ringing

ii. We need to characterise the dynamic structural behaviour of the bell tower

iii. We need to combine the bell-induced forces with the dynamic tower structural characteristics

Bells come in a wide variety of shapes and sizes

Bell forces depend upon:

- Bell system mass
- Bell system inertia (where the mass is distributed away from the pivot axis)
- Ringing speed
- Bell orientation within tower

A “bell system” comprises all of the rotating elements of the bell, including the bell body, headstock, clapper, wheel, stay and counterbalance

Salford Priors
(“classic” tower)

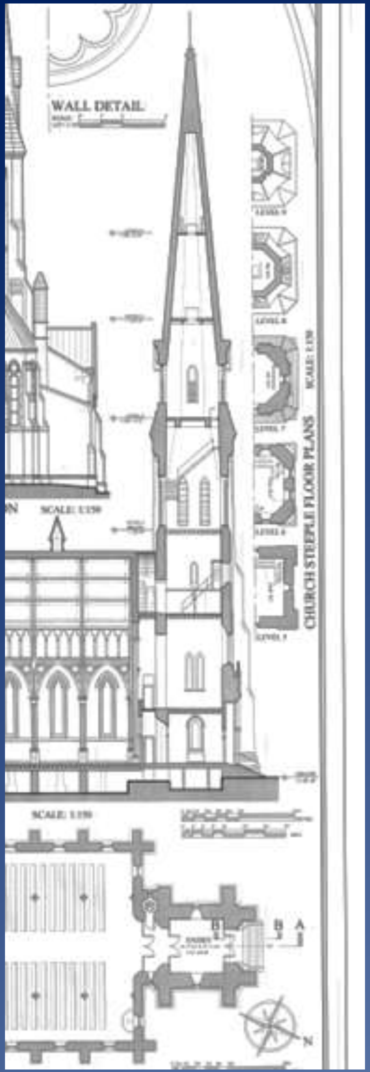
Shotteswell
(steepled tower)

Great Bourton
(isolated tower)

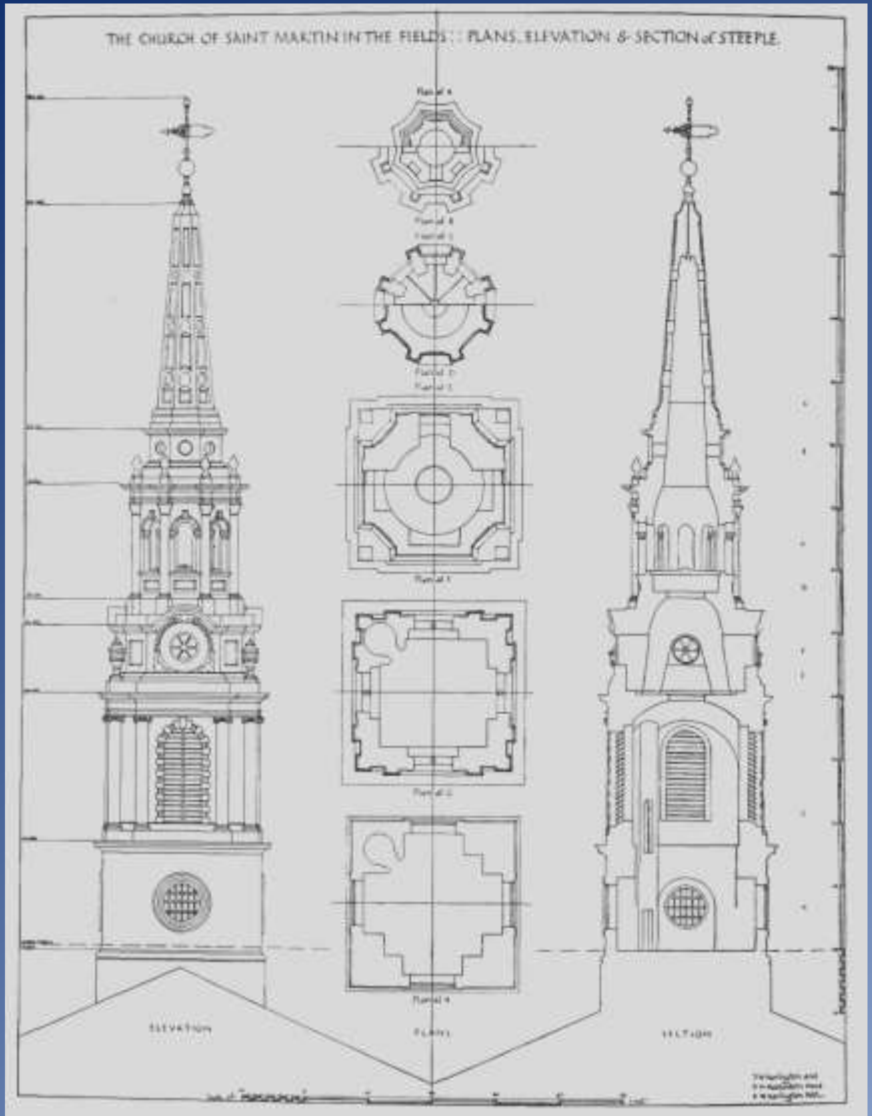
York Minster
(cathedral tower)

Acle
(round tower)

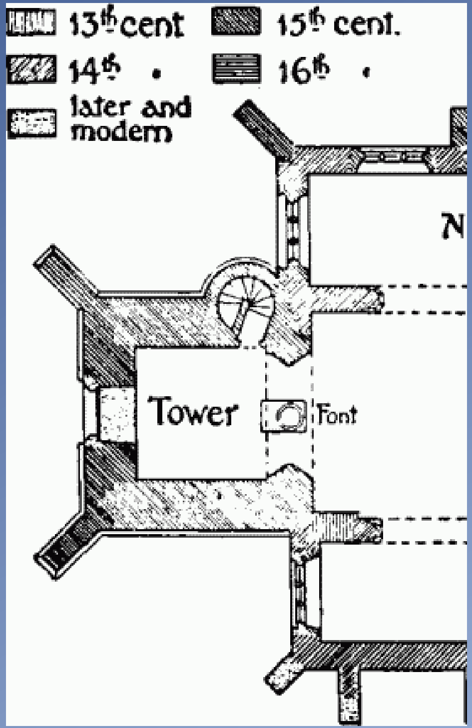
Leamington Spa
(brick tower)



Grace Church, New York



St Martins in the Fields



Potton

Bell tower motion depends upon…

“Obvious Things”

- Time-varying applied Loads (forces and moments) - both magnitude and nature of loads
- Flexibility (or stiffness) of Tower

“Less Obvious Things”

- Mass distribution (along tower height)
- Stiffness distribution
- Centre of twist location
- Distance of mass away from centre of twist
- Natural Frequencies
- Damping

- The problem is complicated, and one cannot expect simple solutions

Predicting bell tower motion

Mathematical models of:

i. Time-varying bell forces
ii. Structural dynamic parameters of bell tower
iii. Dynamic response that combines i) and ii)

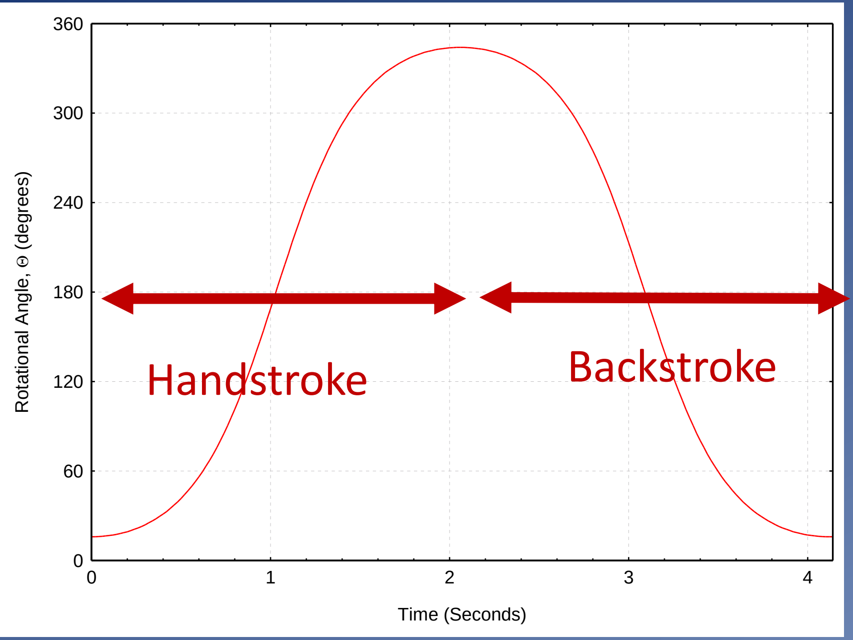
4/8. Theory

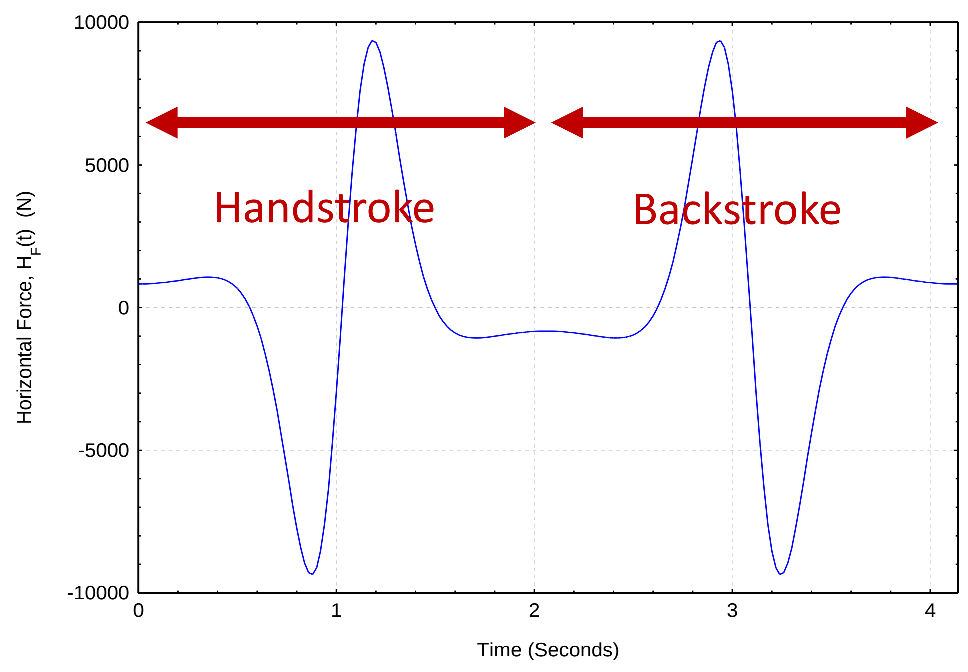
i) Time-varying bell forces $H_F(t)$

(Horizontal)

$$H_F(t) = Mg \times c \times \left(-\left(\frac{3}{2}\right) sin(2\Theta) + 2sin(\Theta)cos(\Theta_0) \right)$$

$$H_F(t) = \frac{4\pi^2}{\tau_0^2}(Mh) \times \left(-\left(\frac{3}{2}\right) sin(2\Theta) + 2sin(\Theta)cos(\Theta_0) \right)$$





Wingrave 8 Bell at Peal Ringing Speed

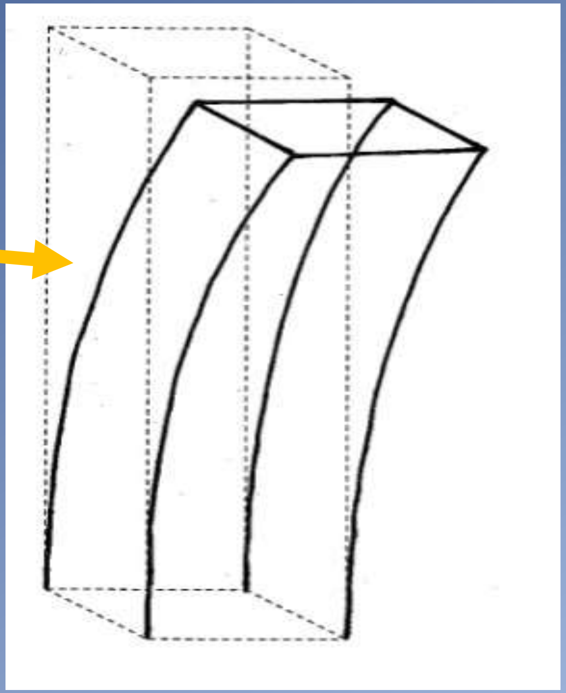
ii) Structural dynamic parameters of bell tower

Natural frequency f_0 (or time period T) of tower .....$f_0 = 1/T$

Mass m, or stiffness k of tower .... $k = (2\pi f_0)^2 m$

Mode shape of vibration associated with f_0

Damping, c ...usually expressed as % of critical damping



Cantilever, 1st Mode bending

iii) Dynamic response

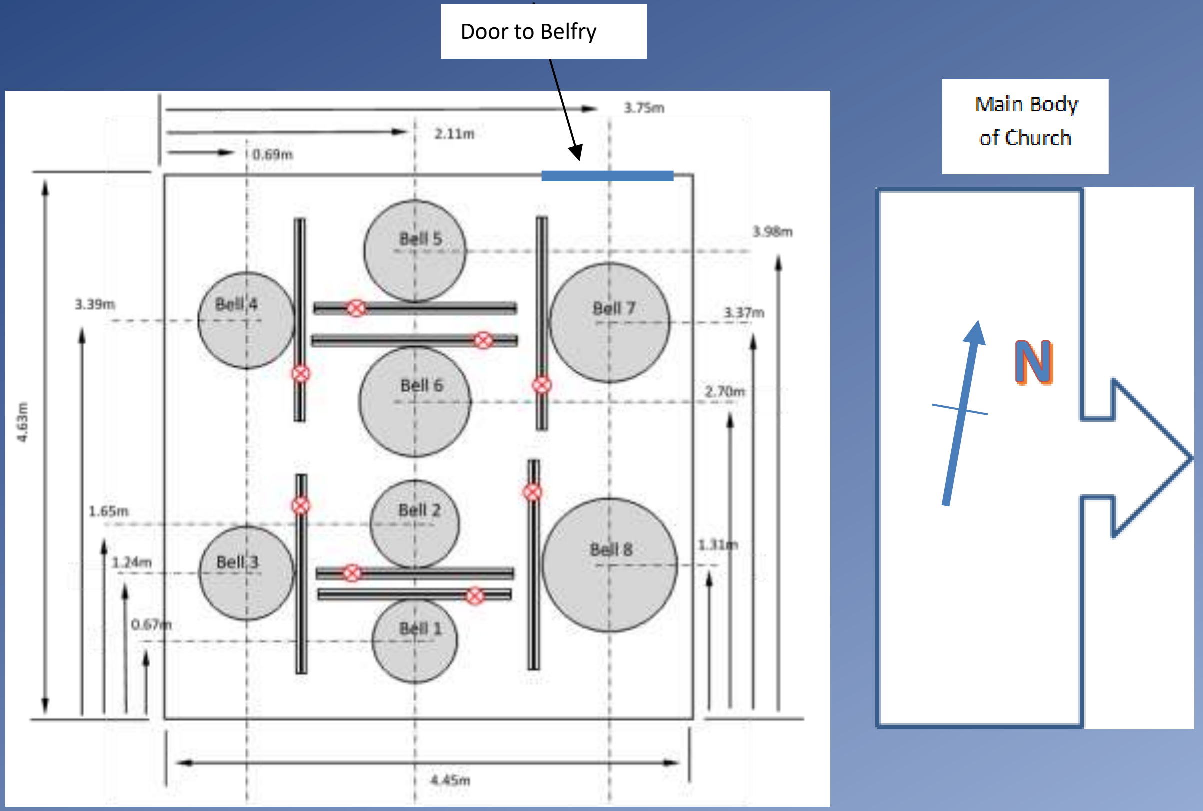
Force, $H_F(t)$ = (acceleration x m) + (velocity x c) + (displacement x k)

Velocity = rate of change of displacement

Acceleration = rate of change of velocity

If we know $H_F(t)$, m, c and k, we can solve the equation above to find the time-varying displacement, velocity and acceleration of the tower.

5/8. Static and dynamic testing at Wingrave Church



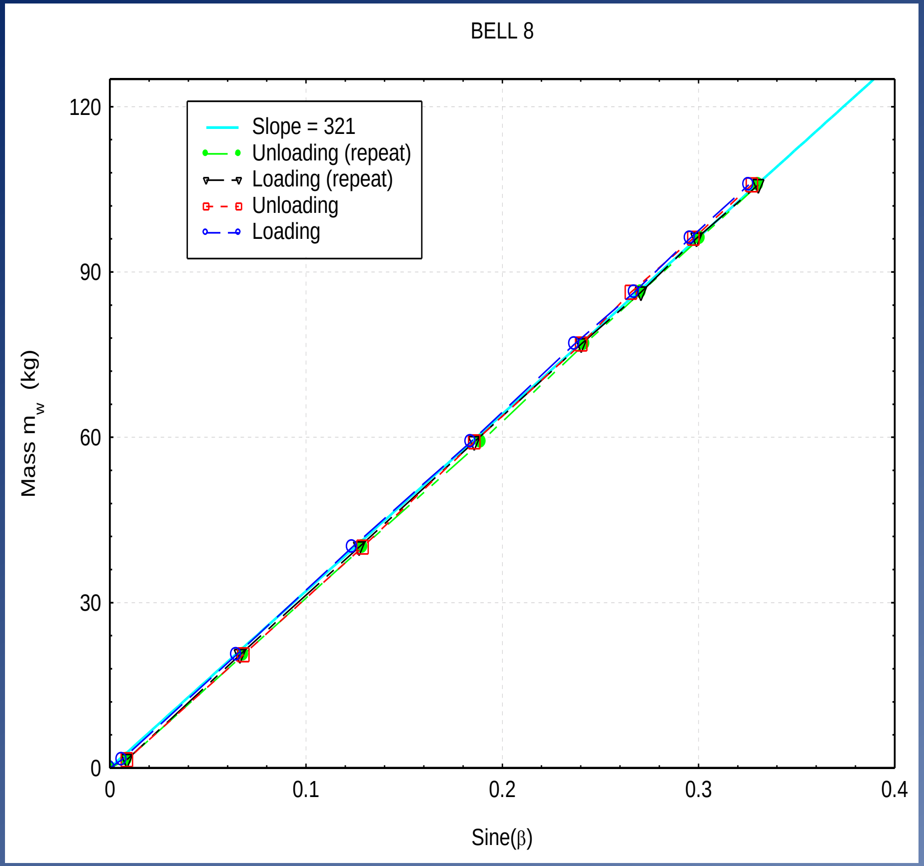
8 Bell (tenor)

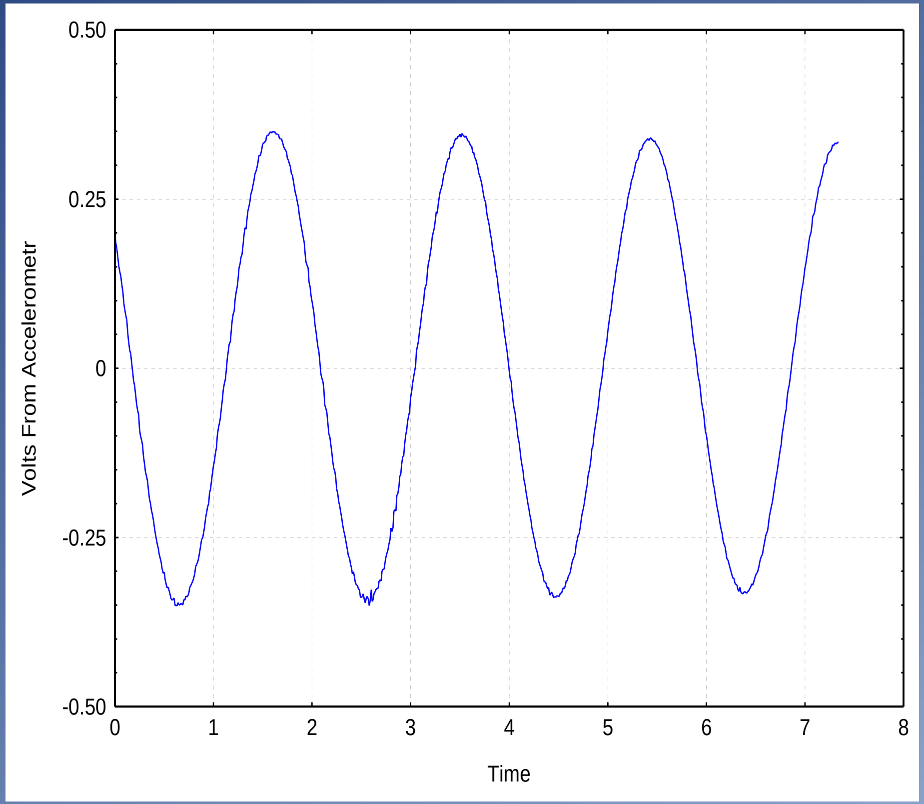
Mears and Steerbank Whitechapel, 1900

Cast weight : 14-2-12, 742 kg

External diameter 445/8”, 1.135m

Static Testing of 8 Bell (tenor)

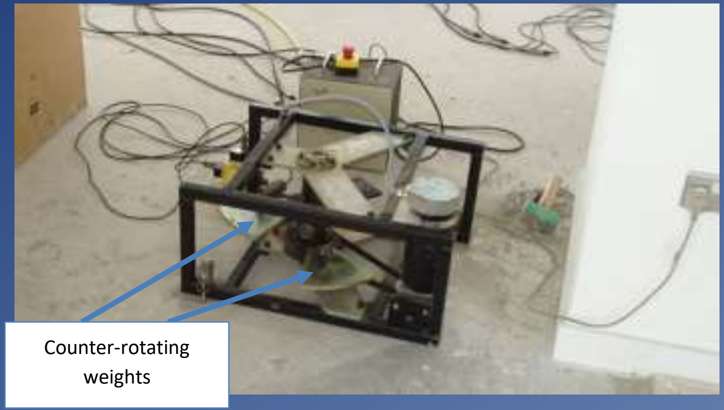




τ_0 = 1.91 seconds

Mh = slope x R = 290.0 kgm

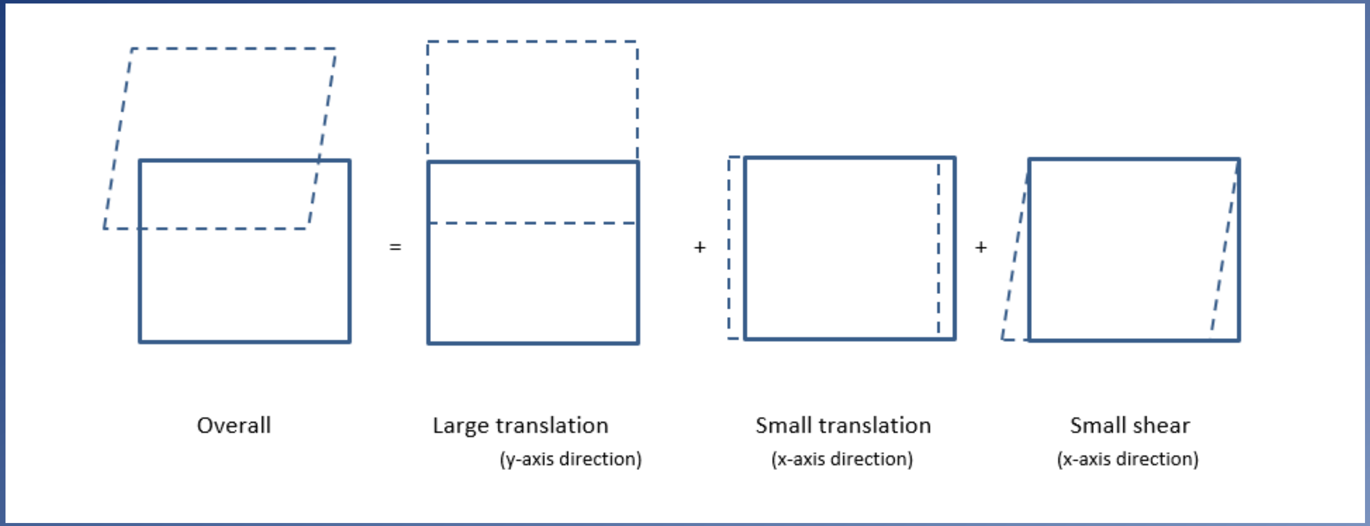
Dynamic Testing

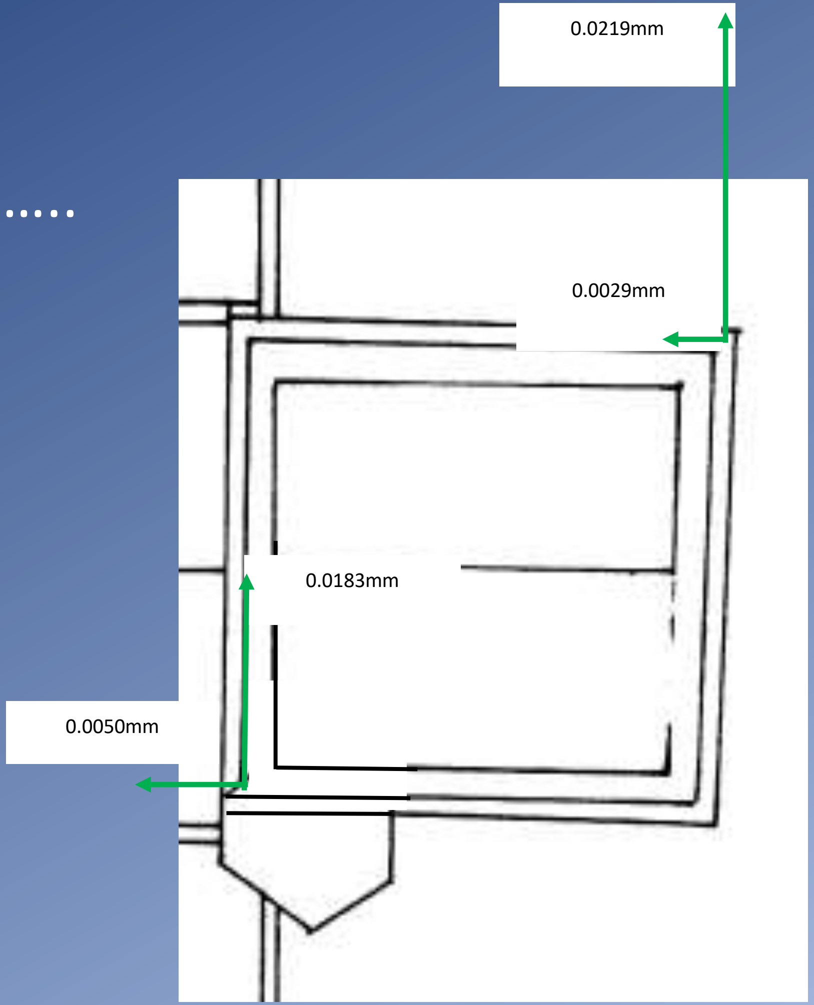


Dynamic Testing:

Preliminary testing to find greatest tower response…..

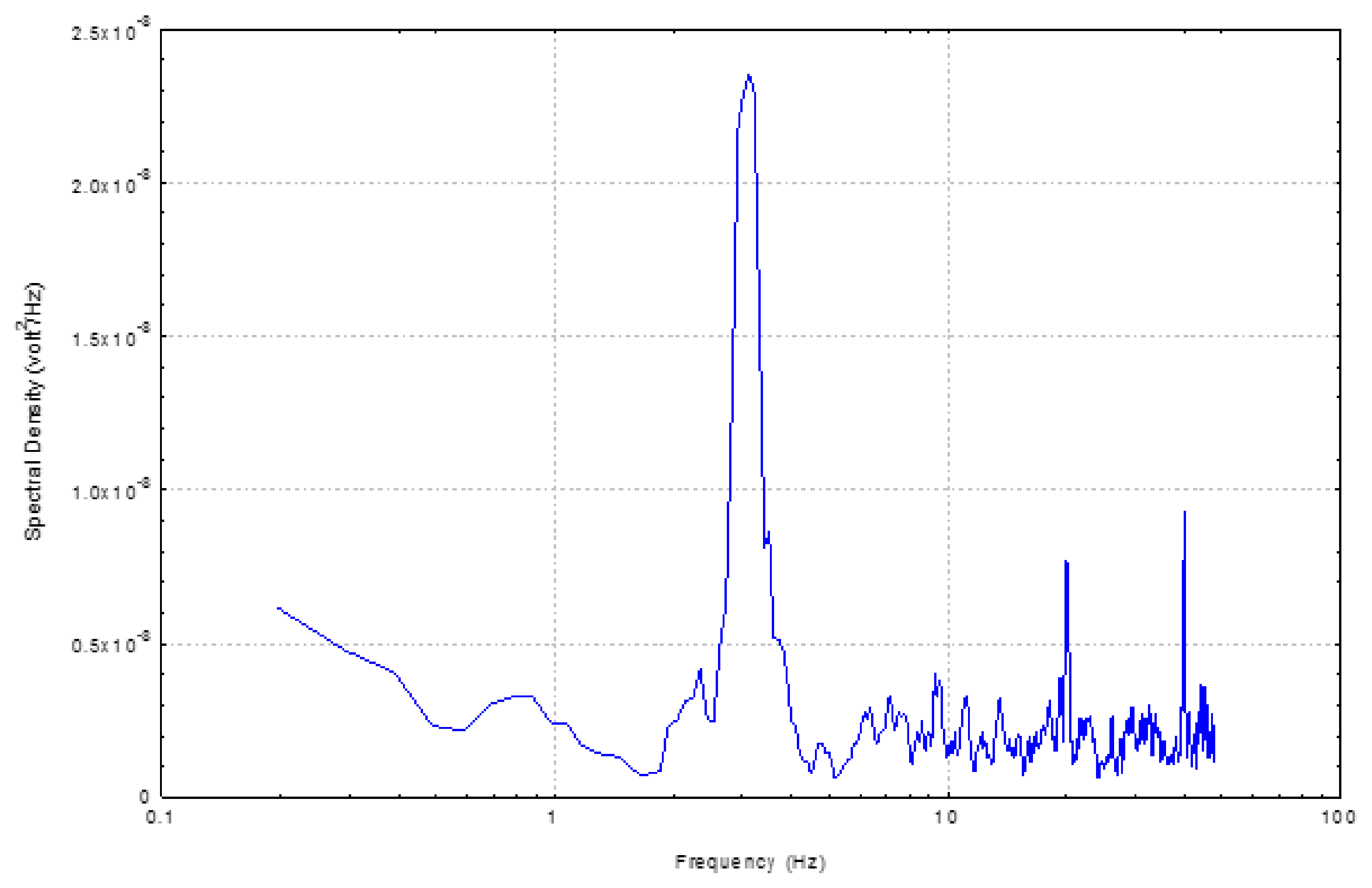
– a surprise!





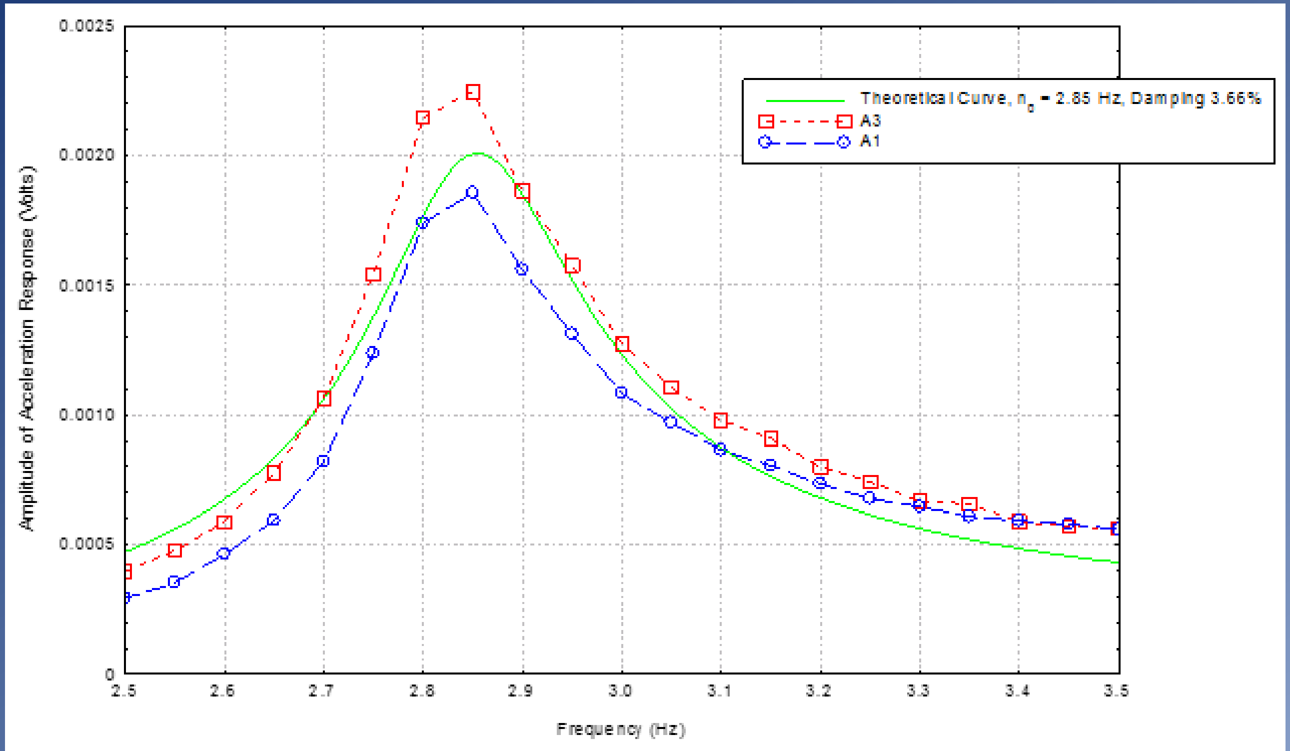
Dynamic Testing

"Ambient" results



Dynamic Testing

Frequency Sweep (N-S direction)

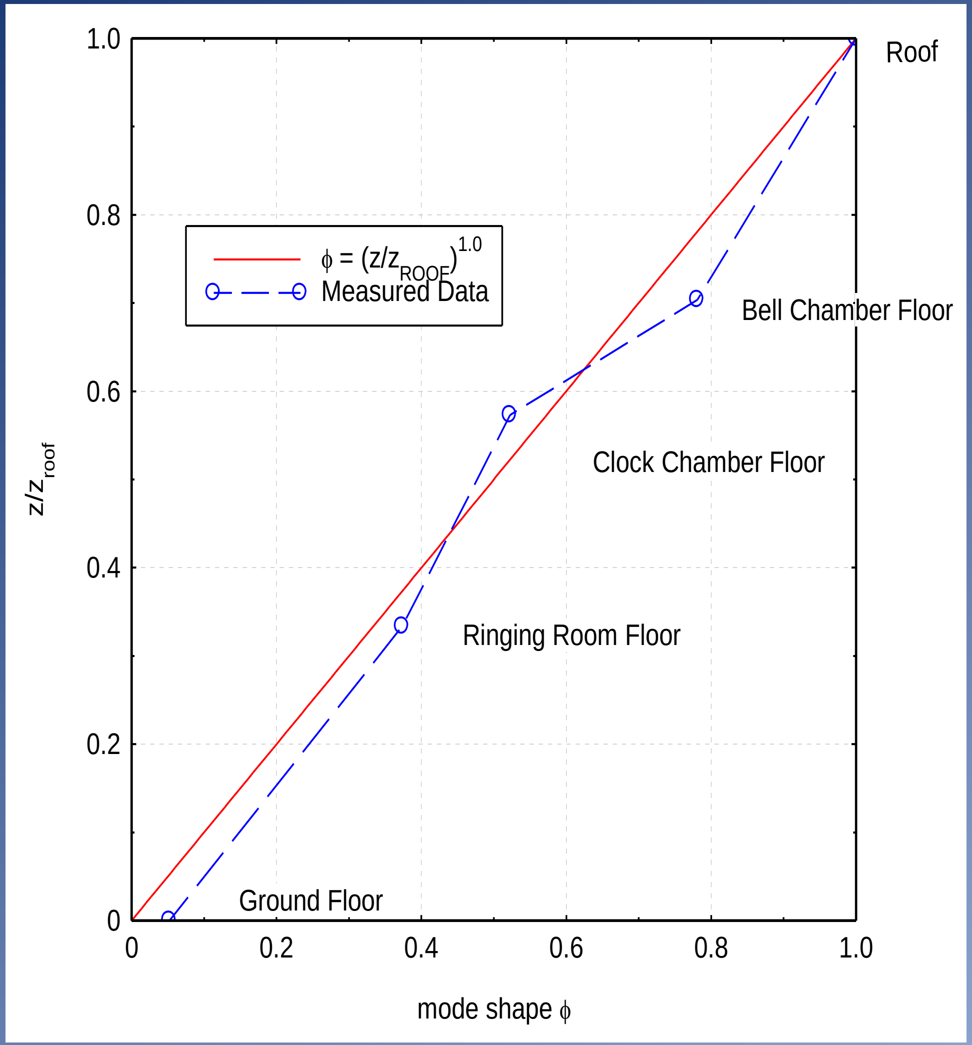


(gives natural frequency f_0, damping c, and resonant amplitude

Since we know applied force amplitude, we can determine building mass m and stiffness k

Dynamic Testing

Mode shape

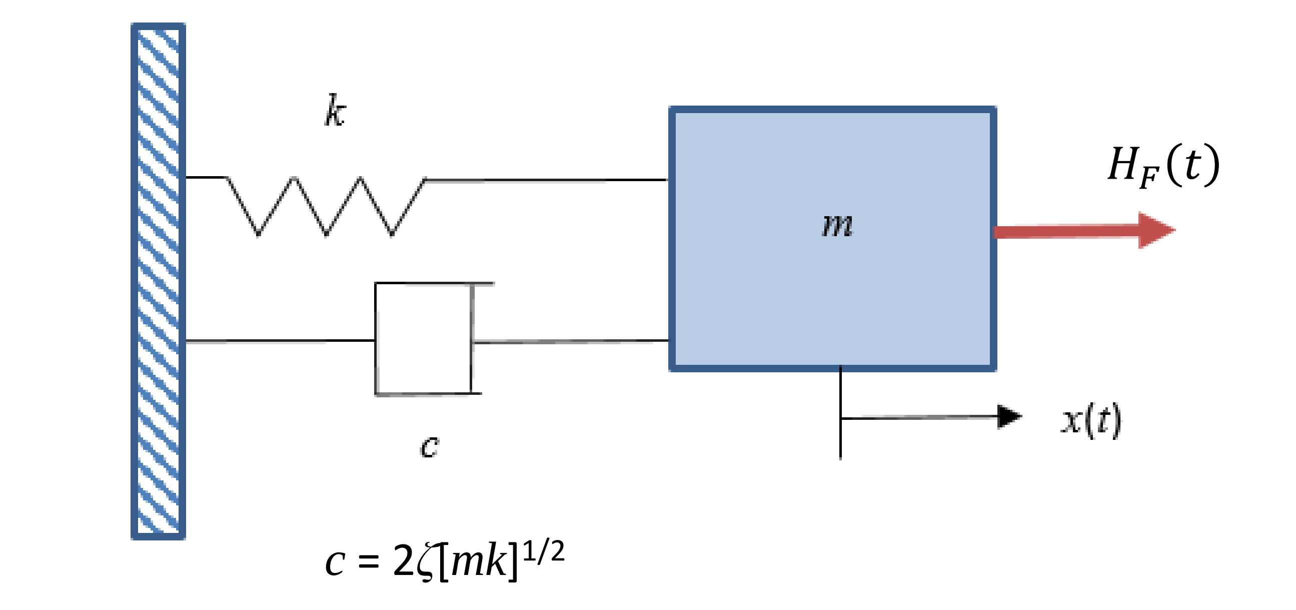


The dynamic structural parameters of the church tower have been determined. Therefore all of the dynamic input parameters needed for the mathematical model are known, viz:

Dynamic Structural Parameters (obtained from shaker testing)

- Natural frequency f_0 — 2.85 Hz
- Mass, m — 293,500 kg
- Spring stiffness, k — 9.24×10^7 N/m
- Critical Damping Ratio, ζ — 3.7%
- Mode shape motion is predominantly in North/South direction
- Mode Shape varies linearly with height above the ground

6/8. Comparison Between Theory and Experiment (Accelerations)

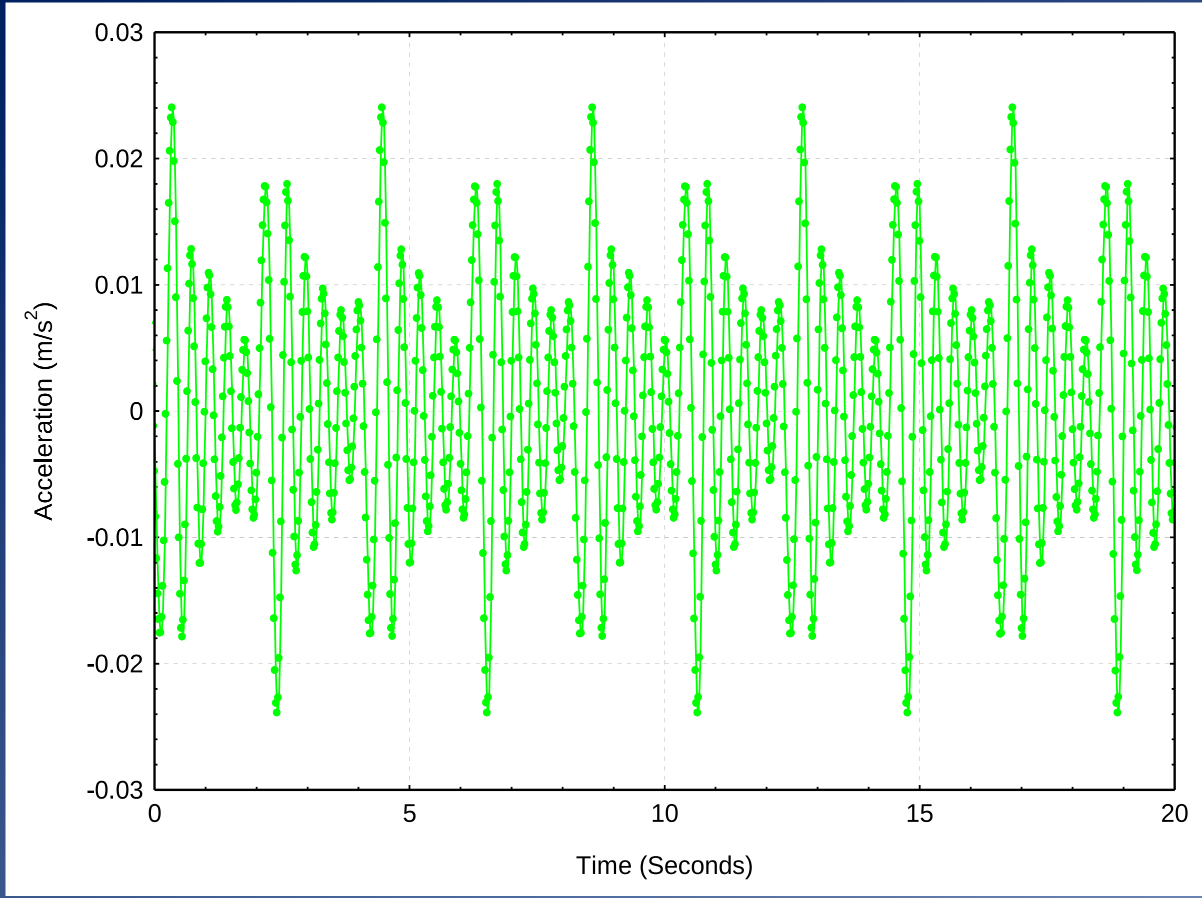


Remembering:

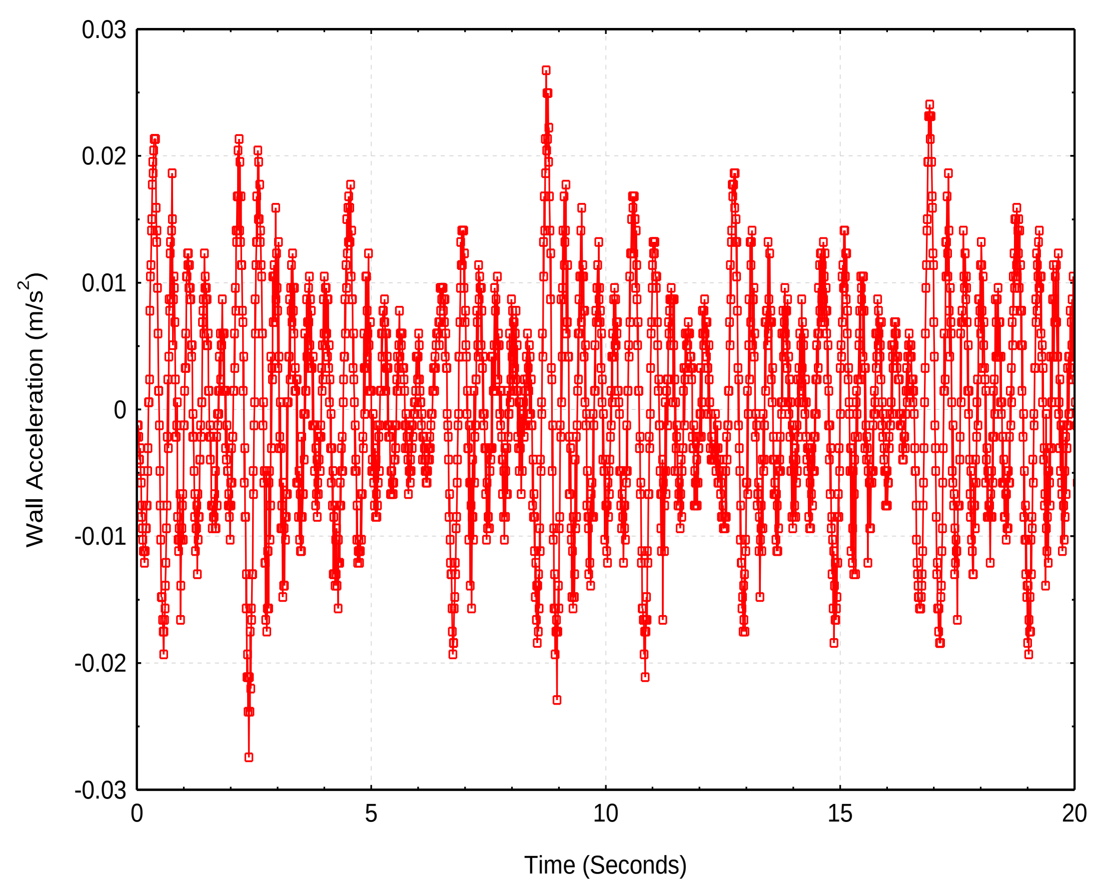
Bell-induced force, $H_F(t)$ = (acceleration x m) +(velocity x c) +(displacement x k)

We know $H_F(t)$, m, k, c, and can use time stepping method to find time-varying displacement $x(t)$, velocity $\dot{x}(t)$ and acceleration $\ddot{x}(t)$

Comparison Between Measured and Predicted Response (Accelerations)

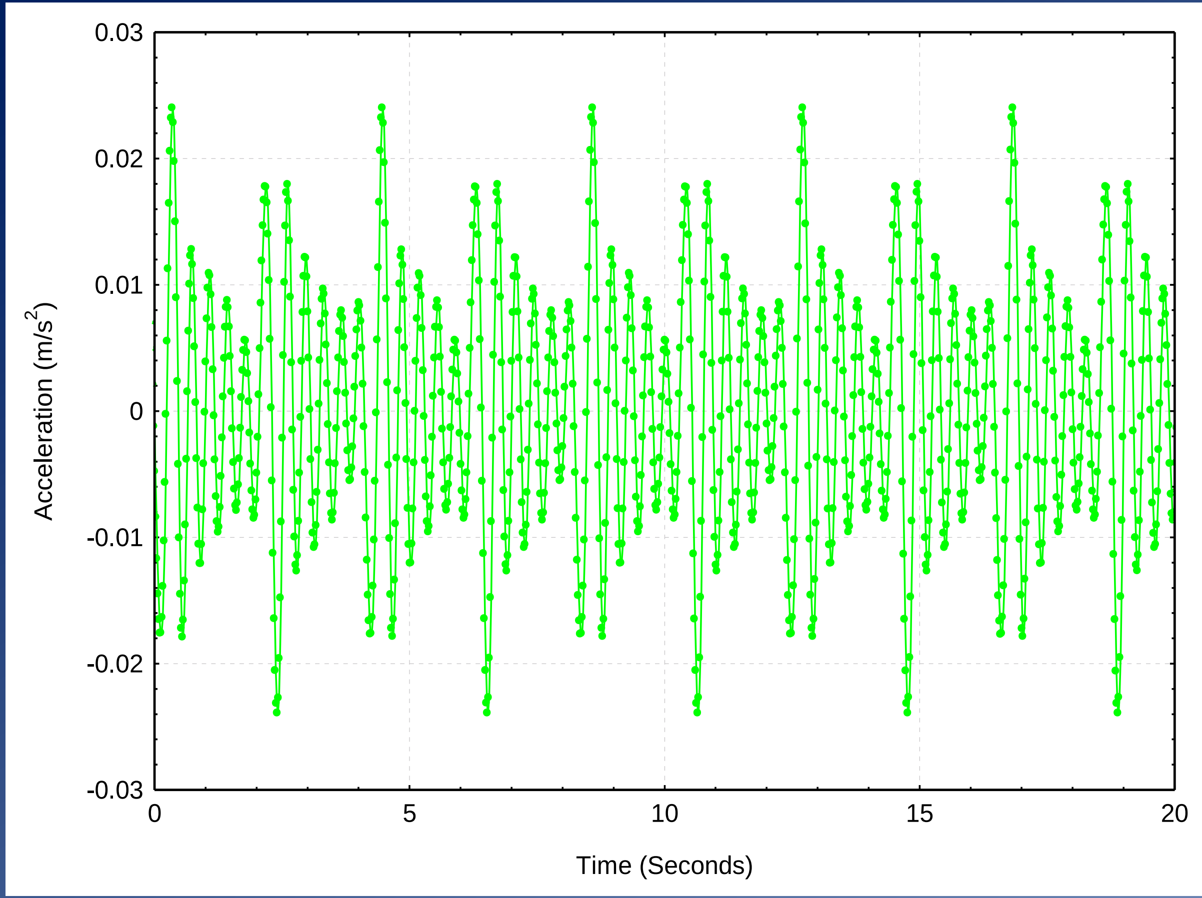


Predicted steady state acceleration

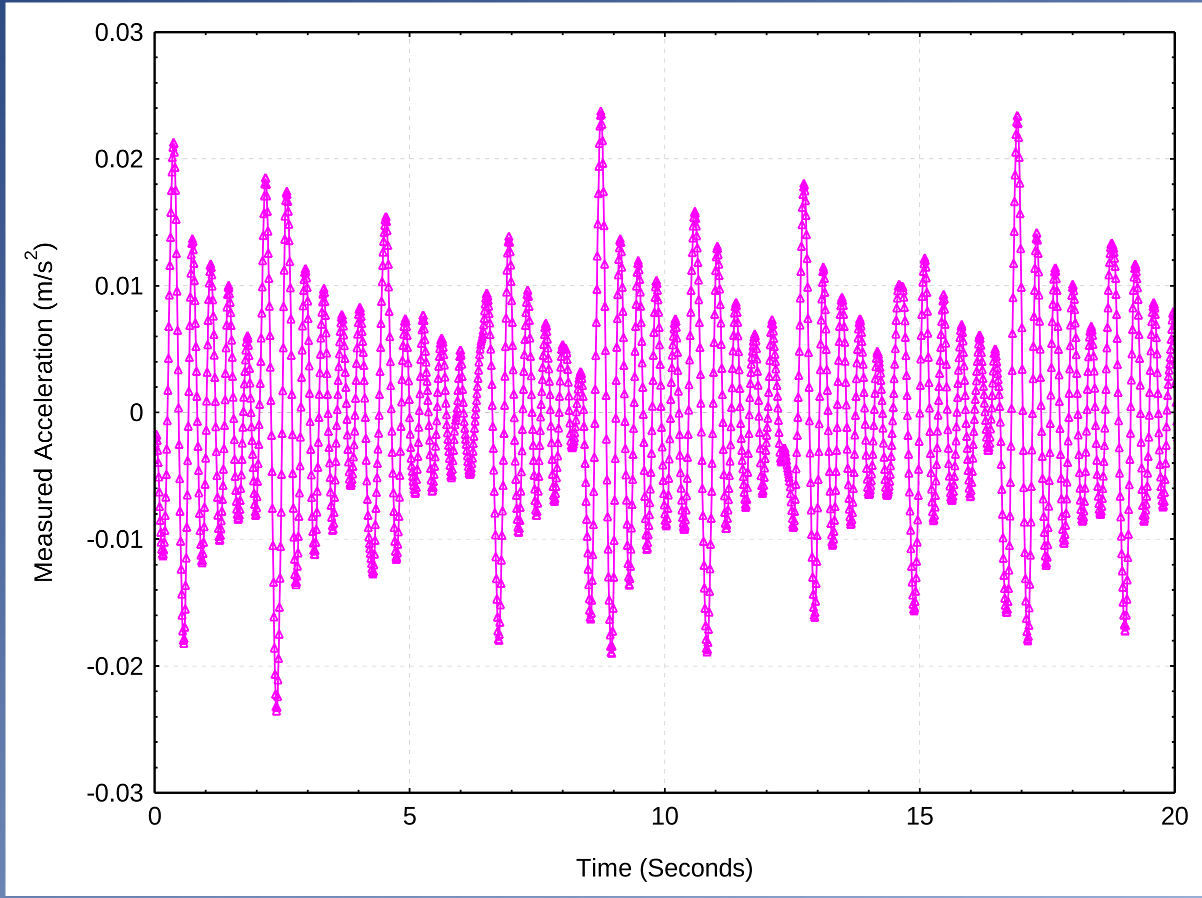


Measured steady state acceleration

Comparison Between Measured and Predicted Response (Accelerations)



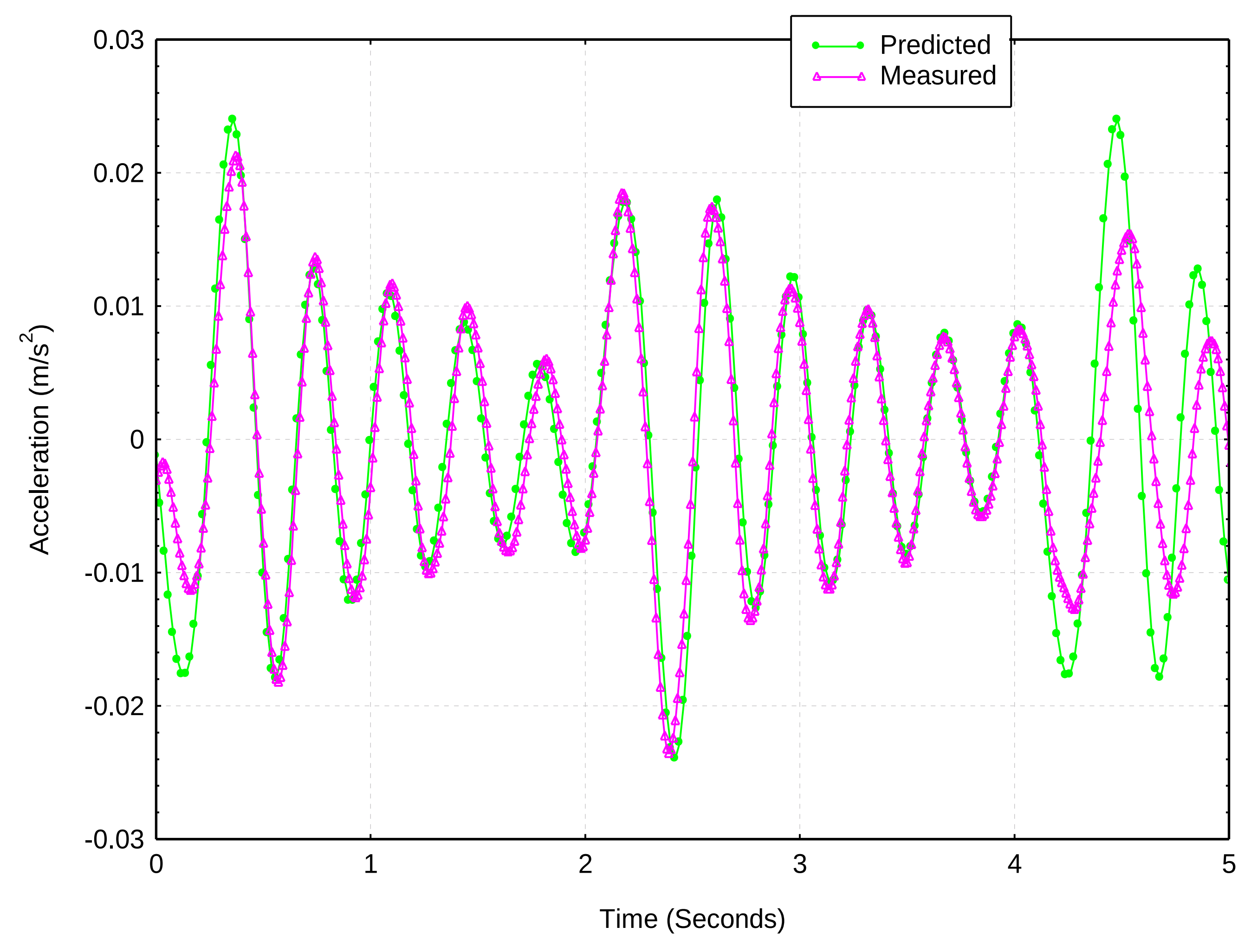
Predicted steady state acceleration



Measured steady state acceleration

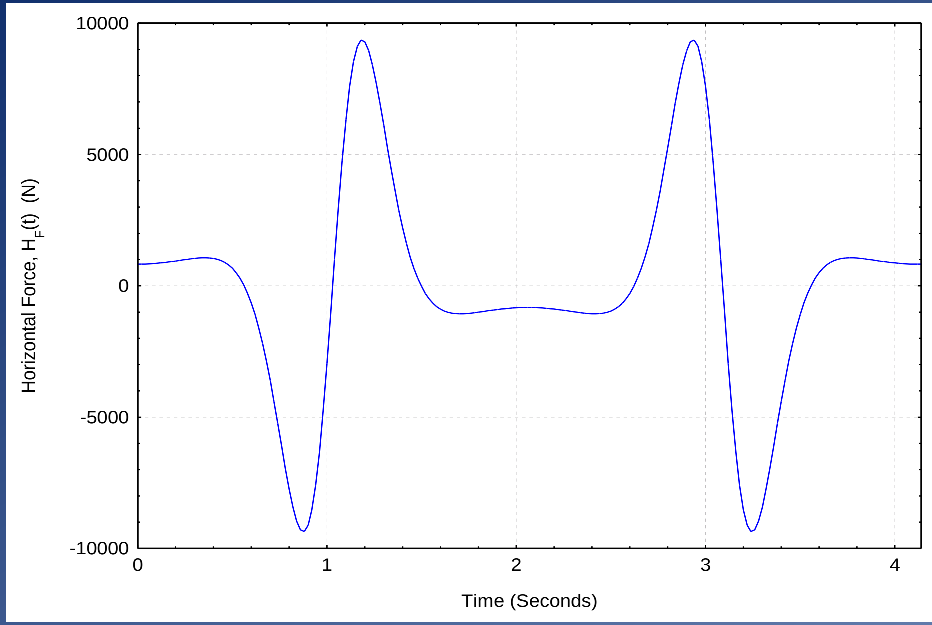
(Low pass filtered at 10 Hz)

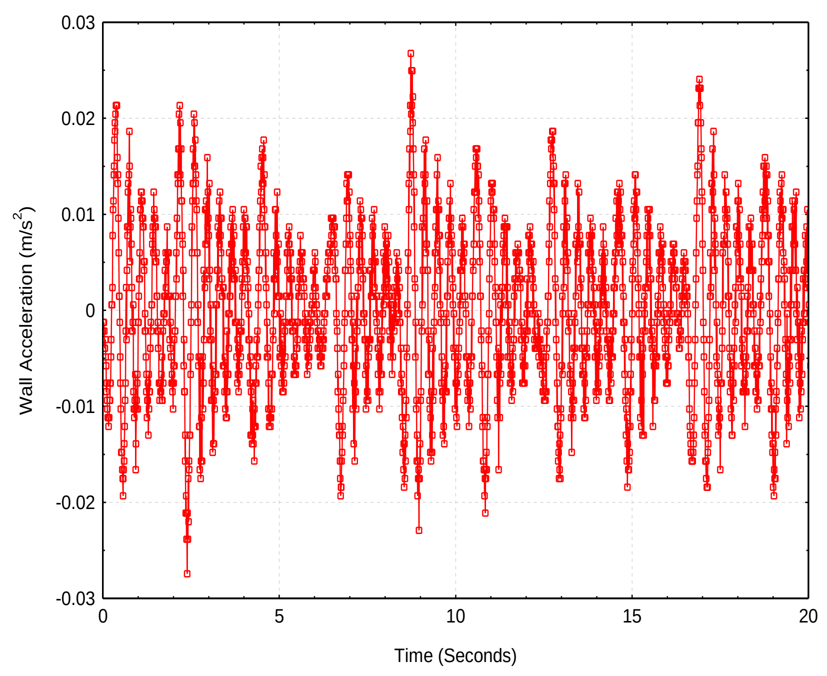
Comparison Between Measured and Predicted Response (Accelerations)



7/8. Using a Bell to Determine Tower Structural Parameters

8 Bell rung at peal ringing bell cycle period of 4.12 seconds (set by metronome)



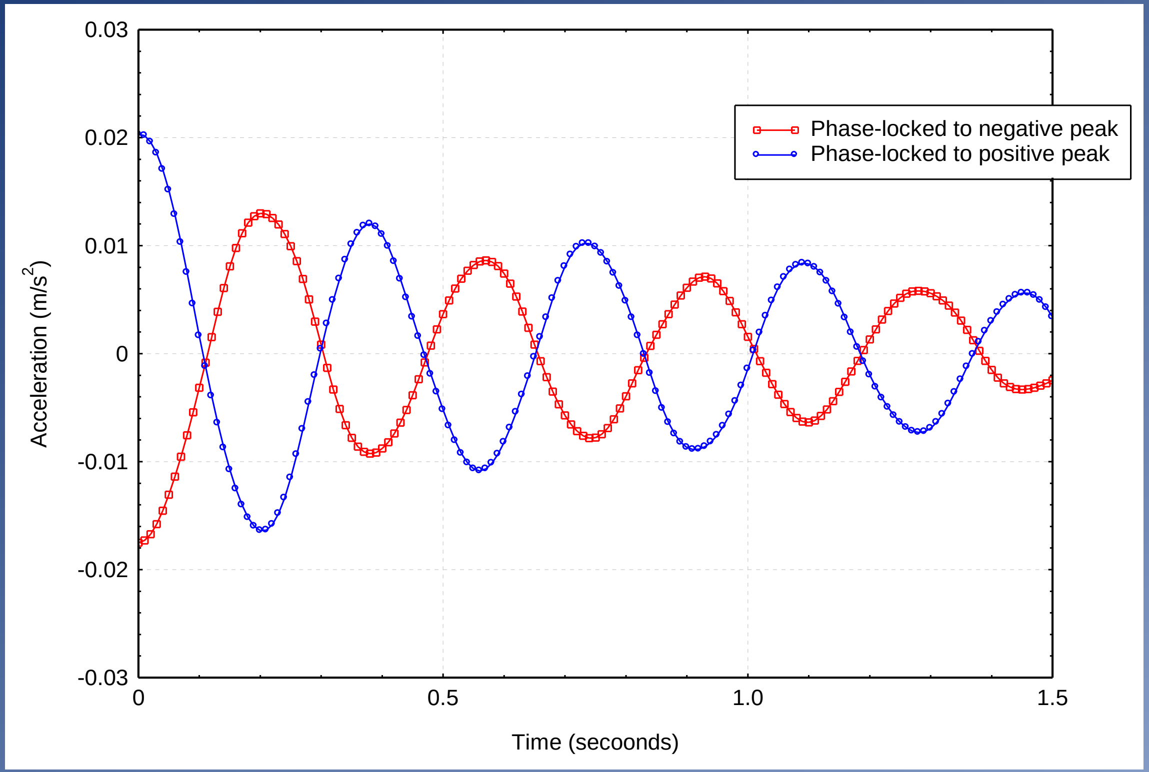


Predicted bell-induced Force (using static loading)

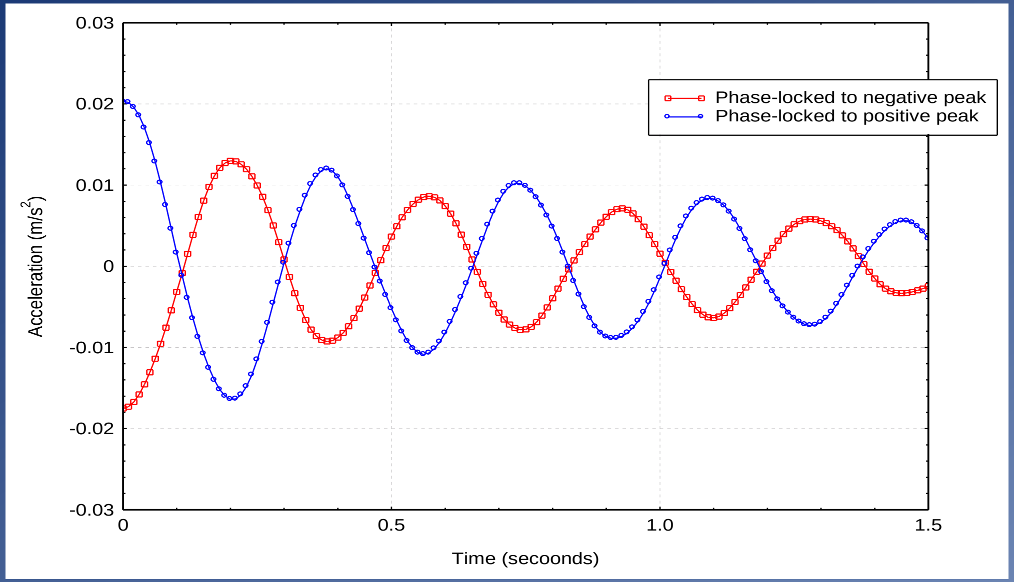
Measured acceleration

Using a Bell to Determine Tower Structural Parameters

Phase-locking and ensemble averaging used to determine characteristic measured acceleration response



Using a Bell to Determine Tower Structural Parameters



Natural frequency f_0 can be found from time interval between respective peaks

Damping can be found from decay rate of peaks

f_0 = 2.80 Hz (2.85Hz, shaker testing)

Critical Damping Ratio, ζ = 3.1% (3.7% shaker testing)

Using a Bell to Determine Tower Structural Parameters

"Trial and Error" method - tower mass found by guessing the mass and calculating the difference between predicted and measured accelerations. Minimising this difference gives the value predicted tower mass.

Bell-induced force, $H_F(t)$ = (acceleration x m)+ (velocity x c) + (displacement x k)

$k = (2\pi f_0)^2 m$

Predicted mass is 299400 kg (shaker testing 293500kg)   - 2% difference !

Using a Bell to Determine Tower Structural Parameters

	Bell Testing	Shaker Testing
Natural frequency, f_0	2.80 Hz	2.85Hz
Critical Damping Ratio, ζ	3.1%	3.7%
Predicted mass m is	299400 kg	293500kg

8/8. Conclusions

- For the Wingrave 8 Bell ringing at peal ringing speed, there is very good agreement between predicted tower response and measured response

- The accelerations measured when the 8 Bell was ringing can be used to predict the tower dynamic structural parameters.

- The bell ringing method can be used to determine the dynamic response of any church tower, without the need for specialist test equipment.